Index

$\Gamma^c{}_{ba}$, 12-2
∇, 12-1
    action on 1-forms, 12-5
    action on functions, 12-5
    action on tensors, 12-6
g_{ab}, 1-7, 18-2
    dynamics of, 1-9, 18-2
k-form, 8-1
1-form, 6-1

acceleration, 2-6
    orthogonal component, 2-16
action for the gravitational field, 18-2
affine parameter, 13-2
algebra
    Grassmann algebra, 8-2
    Lie algebra, 9-6, 13-3
angle between vectors, 15-2
arc length, 1-3, 2-4
atlas, 4-3
    maximal, 4-3
axioms of separability, 3-2

basis, 3-4
    change of, 6-3
    coordinate, 7-3
    dual, 6-4
    local, 3-4
Betti number, 10-14
Bianchi
    identities, 14-12
binormal vector field, 14-5
black hole, 18-12, 19-1
    collision, 19-1
boundary
    of a manifold, 10-5
    operator, 10-14
Brower's theorem, 3-7, 3-9

calculus
    exterior, 8-1
    vector calculus, 8-6
Cartesian product
    of vector spaces, 8-2
chart, 4-3
Christoffel
    symbols, 15-5
closed forms, 8-9
co-chain, 10-12
cohomology, 3-5, 10-12
    de Rham cohomology, 10-12
commutator, 7-2
    geometric interpretation, 7-5
compactness
    as topological invariant, 3-5
congruence, 11-2
connected, 3-2
connectedness
    as topological invariant, 3-6
connection
    antisymmetric part, 14-3
    coefficients, 12-2
    linear, 12-1
    metric, 15-4
      uniqueness, 15-5
    Newtonian, 13-7, 16-4

non-uniqueness, 12-2
symmetric, 14-1
symmetric part, 13-4
transformation of coefficients, 12-3
conservation laws, 17-4
continuity equation, 16-3
continuous media, 17-4
contraction
Lorentz, 1-5
of indices, 6-5
contravariant, 6-4
convective derivative, 17-4
coordinate
basis, 7-3
function, 4-6
coordinate system, 4-4
adapted, 11-2
coordinate transformation, 1-6
coordinates
Eddington-Finkelstein, 18-10
Galilean, 16-1
Kruskal, 18-12
normal, 13-3
covariant, 6-4
derivative, 12-1
covector, 6-1
cube in a manifold, 10-2
curl, 8-7
curvature, 2-1, 14-1
for different dimensions, 14-12
Gaussian, 2-19
geometric interpretation, 14-8–14-11
intrinsic, 14-9
mean, 2-19
normal, 2-16
of $\mathbb{R}^2$, 15-11
of $\mathbb{S}^2$, 15-8
of a curve, 2-7
pincipal, 2-17
relative, 5-6
tensor, 14-3
calculation of, 14-4
curvature function, 14-5
curve, 5-1
acceleration of, 2-6
arc length, 2-4, 2-5
complete, 11-2
geodesic, 13-1
in $\mathbb{E}^3$, 2-2
in $\mathbb{E}^3$, 2-1
integral, 11-2
integral curve, 11-1
on a manifold, 5-1
plane, 2-9
regular, 2-3
reparametrisation, 2-3, 2-5

derivative
as a partial derivative, 11-5
convective, 17-4
covariant, 2-12, 12-1
directional, 2-11
exterior, 8-3
Lie, 11-3–11-8
Lie derivative, 11-1
Lie derivative
geometric interpretation, 11-4
Lie, of n-forms, 11-7
Lie, of 1-form fields, 11-6
Lie, of functions, 11-3
Lie, of tensors, 11-8
Lie, of vector fields, 11-4
diffeomorphism, 4-4
differentiable
function, 4-6
manifold, 4-1, 4-3
differential
form, 6-1
map, 9-2
of a function, 6-1
differential forms, 8-1
differential of a function, 6-1

dimension, 4-1
  as topological invariant, 3-6
direct sum, 8-2
distance
  proper, 1-4
divergence, 8-8
Doppler effect, 1-7
drag along curves, 11-3
dynamics
  determined by geometry, 2-16

Eddington-Finkelstein
  coordinates, 18-10
Einstein
  equations, 18-1, 18-4, 18-7
  tensor, 15-6
Einstein notation, 1-7, 6-1
electrodynamics, 16-1
energy conditions
  dominant, 18-18
  strong, 18-18
  weak, 18-18
energy-momentum pseudo-tensor, 19-6
energy-momentum tensor, 17-7
  electromagnetic, 17-3
  of a perfect fluid, 17-4
equivalence
  topological, 3-1
equivalence principle, 18-1
Euler, 2-19
  equation, 17-5
Euler's Formula, 2-19
exact forms, 8-9
exterior
  derivative, 8-3
  product, 8-2
exterior calculus, 8-1

field
  gravitational, 1-1, 1-9
  tensor field, 7-1
  vector field, 2-6, 7-1
first fundamental form, 15-13
fixed point problem, 3-7, 3-9
fluid
  flow, 17-5
  isentropic, 17-5
  perfect, 17-5
fluid dynamics, 16-1, 16-3
form
  closed, 10-12
  differential, 6-1
  exact, 10-12
  integrable, 10-1
form operator, 2-13
forms
  closed, 8-9
  differential, 8-1
  exact, 8-9
free fall, 16-4
Frenet's formulae, 2-1, 2-7
Frenet's reference frame, 2-6
Frobenius's theorem, 12-7
function
  continuous, 3-5
  differentiable, 4-6
  distance function, 3-9
  distortion function, 2-21
  integrable, 10-1
  structure function, 7-3
future, 16-1

Galileo's principle, 1-9
Gauss' theorem, 10-10
Gaussian curvature, 2-19
geodesic, 13-1
  minimum-action principle, 15-19
  time-like, 17-1
  uniqueness, 13-1, 13-2
geodesics
  in the Schwarzschild metric, 18-13
  on $\mathbb{T}^2$, 15-13

geometric units, 1-10
geometry
    of curved spaces, 1-1
gradient, 6-1, 6-3, 8-7
Grassmann algebra, 8-2
gravitation, *see* general relativity and metric tensor
    linear theory, 19-1
gravitational
    radiation, 19-1
gravitational field, 1-1
gravitational potential, 16-4
gravitational redshift, 1-8
gravitational wave, 19-7
    radiated power, 19-8
group
    cohomology group, 10-12
    homology group, 10-14
    Lie group, 9-6, 13-3, 18-6

Hamilton
    equations, 16-5
Hamiltonian dynamics, 16-5
helix, 2-2
    hyperbolic, 2-2
hole
    black, 18-12
    white, 18-12
homeomorphism, 3-5
homology, 3-5, 10-12

index
    of vector field, 3-7
inner product, 8-7, 15-1
integrable form, 10-1
integrable function, 10-1
integral
    integration on manifolds, 10-1
integral of an n-form, 10-2
integration
    on manifolds, 10-1
isentropic flow, 17-5, 17-6

Jacobi identity, 7-2

Killing
    equations, 18-6
    vector, 18-6
Kruskal coordinates, 18-12

Lagrangian density, 17-7
Leibnitz's rule, 5-5
length
    of a curve, 15-2
    of arc, 1-3
Levi-Civita
    connection, 15-5
    tensor, 17-2
Lie
    algebra, 9-6
    group, 9-6
    Lie group, 18-6
Lie algebra, 13-3
Lie derivative, 11-1, 11-3, 11-4, 11-6
    as partial derivative, 11-5
    geometric interpretation, 11-4
    of a tensor field, 11-8
    of an n-form, 11-7
Lie group, 13-3
light cone, 15-6
local causality condition, 18-19
Lorentz
    contraction, 1-5
    transformation, 1-1
Lorentzian manifold, 17-1

Möbius strip, 4-4, 9-5
magnitude of a vector, 15-2
manifold
    differentiable, 4-1, 4-3
    Galilean, 16-1
    Lorentzian, 15-3
    orientable, 8-9

pseudo-Riemannian, 15-3
Riemannian, 15-3
with boundary, 10-4
map
between manifolds, 9-1
differential, 9-2
exponential, 13-3
mass moment
quadrupolar , 19-5
Maxwell
equations, 17-1
tensor, 17-1
Maxwell relations, 16-7
mean curvature, 2-19
mechanics
analytical, 8-10
Newtonian, 16-1
metric, 15-3
coefficients, 15-3
Euclidean, 16-1
Lorentzian, 15-3
of $\mathbb{S}^n$, 15-10
pseudo-Riemannian, 15-1, 15-3
Riemannian, 15-3
Schwarzschild, 18-8
signature, 15-3
minimum-action principle, 15-19
Minkowski space, 15-6
Morse theory, 3-9

neighbourhood, 3-2
Newton
equations, 16-5
Newtonian
connection, 13-7
potential, 13-6
Newtonian mechanics, 16-1
Newtonian potential, 16-4
normal vector field, 14-5
notation
Einstein, 1-7, 6-1
one-form, 6-1
operator
boundary, 10-4, 10-5, 10-12
orientability, 8-10
orthogonality, 12-6

parallel
transport, 12-2
partition of unity, 10-3
past, 16-1
perfect fluid, 17-5
Poincaré
group, 17-1
potential
gravitational, 16-4
Newtonian, 16-4
principal direction, 2-17
principle of relativity, 18-1
product
inner, 8-7, 15-1
tensor product, 6-5
vector product, 8-8
proper distance, 1-4
proper time, 1-3, 17-1
pull-back, 9-2

quadrupolar mass moment, 19-5

radiation
gravitational, 19-1
rate of change, 5-1
reference frame
Frenet, 2-6
inertial, 1-1
relative curvature, 5-6
relativity
general, 1-1, 18-1
principle, 18-1
principle of, 1-1
special, 1-1, 17-1
reparametrisation, 2-3
Ricci

curvature, 14-13
identity, 14-10
scalar, 14-12, 15-6
tensor, 14-12
Riemann
tensor, 14-3
geometric interpretation, 14-10, 14-11
in Newtonian space-time, 16-4
Riemann tensor
for the Schwarzschild metric, 18-13

Schild's argument, 1-9
Schwarzschild
geodesics, 18-13
horizon, 18-11
metric, 18-8
radius, 18-10
separability axioms, 3-2
set
closed, 3-2
open, 3-2
simultaneity, 16-1
singularity, 18-10
space
connected, 3-2
curved, 1-9
dual, 6-1
locally Euclidean, 4-1
metric, 3-9
of vector fields, 7-3
tangent, 5-1, 5-3
topological, 3-1
T_0, 3-2
T_1, 3-3
T_2, 3-3
connected, 3-2
first-countable, 3-4
Frèchet, 3-3
Hausdorff, 3-3
homeomorphic, 3-5
Komolgorov, 3-2
second-countable, 3-4
stereographic projection, 4-1
Stokes theorem, 17-3
Stokes' theorem, 10-5, 10-14
in $\mathbb{R}^3$, 10-10
surface
in $\mathbb{E}^3$, 2-11
symplectic
differential form, 16-5
geometry, 16-5
manifold, 16-5
system
dynamic, 5-7
inertial, 16-4
intertial, 17-1
local coordinate system, 4-4
Newtonian, 16-4
thermodynamic, 16-6

tangent bundle, 7-1
tangent space, 5-1, 5-3
tangent vector, 5-1, 5-2
tangent vector field, 14-5
tensor, 6-1, 6-4
analysis, 7-1
antisymmetric, 6-5
electromagnetic, 17-1
energy-momentum, 17-7, 18-5
canonical forms, 18-16
electromagnetic, 17-3
of a perfect fluid, 17-4
field, 7-1
metric
as a gravitational field, 18-2
product, 6-5
rank, 6-5
symmetric, 6-5
theorem
Brower, 3-7, 3-9
thermodynamics, 16-1, 16-6
first law, 17-5

first law of, 17-5
time dilation, 1-3
topological
invariant, 10-14
invariants, 3-5
topological space, 3-1
connected, 3-2
Frèchet, 3-3
Hausdorff, 3-3
Komolgorov, 3-2
topology, 3-1
algebraic, 10-14
discrete, 3-2, 3-3
torsion, 2-1, 14-1
coefficients, 14-2
geometric interpretation, 14-6
of a curve, 2-7
tensor, 14-2
torsion function, 14-5
torus, 4-5
flat torus, 15-16
transformation
coordinate, 1-6
Lorentz, 1-1
transport
Lie transport: *see* Lie derivative, 11-1
parallel, 12-2

umbilical point, 2-17
units
geometric, 1-10

vector
binormal, 2-6
normal, 2-6
null, 15-3
orthogonality, 15-7
space-like, 15-3, 16-1
tangent, 2-6, 5-1, 5-2
time-like, 15-3, 16-1
vector field, 2-6
binormal, 14-5
normal, 14-5
tangent, 14-5
velocity, 5-1
angular, 16-2
of a curve, 2-3
volume element, 8-6

waves
gravitational, 19-1
wedge product, 8-2
Weyl
curvature, 14-13
tensor, 14-12
white hole, 18-12